

Exercise 11

If $x^2 + y^2 + z^2 = 9$, $dx/dt = 5$, and $dy/dt = 4$, find dz/dt when $(x, y, z) = (2, 2, 1)$.

Solution

Differentiate both sides of the given equation with respect to t and use the chain rule.

$$\frac{d}{dt}(x^2 + y^2 + z^2) = \frac{d}{dt}(9)$$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) + \frac{d}{dt}(z^2) = 0$$

$$(2x) \cdot \frac{dx}{dt} + (2y) \cdot \frac{dy}{dt} + (2z) \cdot \frac{dz}{dt} = 0$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} + z \frac{dz}{dt} = 0$$

Solve for dz/dt .

$$\frac{dz}{dt} = -\frac{x}{z} \frac{dx}{dt} - \frac{y}{z} \frac{dy}{dt}$$

If $dx/dt = 5$ and $dy/dt = 4$ and $x = 2$ and $y = 2$ and $z = 1$, then

$$\left. \frac{dz}{dt} \right|_{\substack{x=2 \\ y=2 \\ z=1}} = -\frac{2}{1}(5) - \frac{2}{1}(4) = -18.$$